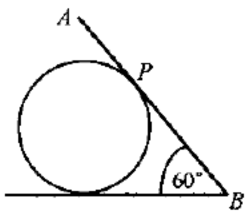
Take $g = 9{\cdot}8\ \text{ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

1. Particles of mass $2m$, $3m$ and $5m$ are placed at the points in the x-y plane with coordinates $(-1, 5)$, $(0, 6)$ and $(3, -2)$ respectively.
Find the coordinates of the centre of mass of this system of particles. [4]

2. A lorry of mass 3800 kg is pulling a trailer of mass 1200 kg along a straight horizontal road. At a particular moment, the lorry and trailer are moving at a speed of 10 ms^{-1} and accelerating at 0·8 ms^{-2}. The resistances to the motion of the lorry and the trailer are constant and of magnitude 1600 N and 600 N respectively.
Find the rate, in kW, at which the engine of the lorry is working. [4]

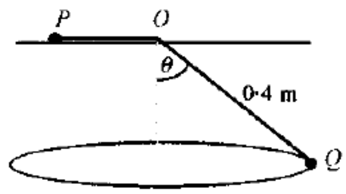
3. A bullet of mass 0·05 kg is fired with speed u ms^{-1} from a gun, which recoils at a speed of $0{\cdot}008u$ ms^{-1} in the opposite direction to that in which the bullet is fired.
   (i) Find the mass of the gun. [2]
   (ii) Find, in terms of u, the kinetic energy given to the bullet and to the gun at the instant of firing. [3]
   (iii) If the total kinetic energy created in firing the gun is 5103 J, find the value of u. [2]

4. 

A uniform plank AB, of mass 3 kg and length 2 m, rests in equilibrium with the point P in contact with a smooth cylinder. The end B rests on a rough horizontal surface and the coefficient of friction between the plank and the surface is $\frac{1}{3}$. AB makes an angle of 60° with the horizontal.
If the plank is in limiting equilibrium in this position, find
   (i) the magnitude of the force exerted by the cylinder on the plank at P, [6]
   (ii) the distance AP. [3]

5. Two smooth spheres A and B have equal radii and masses 0·4 kg and 0·8 kg respectively. They are moving in opposite directions along the same straight line, with speeds 3 ms^{-1} and 2 ms^{-1} respectively, and collide directly. The coefficient of restitution between A and B is 0·8.
   (i) Calculate the speeds of A and B after the impact, stating in each case whether the direction of motion has been reversed. [8]
   (ii) Find the kinetic energy, in J, lost in the impact. [2]

6. A particle P, of mass 0·5 kg, rests on the surface of a rough horizontal table. The coefficient of friction between P and the table is 0·5. P is connected to a particle Q, of mass 0·2 kg, by a light inextensible string passing through a small smooth hole at a point O on the table, such that the distance OQ is 0·4 m. Q moves in a horizontal circle while P remains in limiting equilibrium.



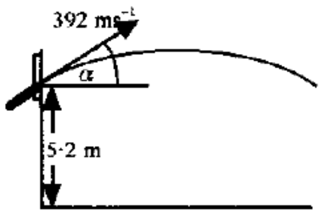
(i) Calculate the angle θ which OQ makes with the vertical. [4]

(ii) Show that the speed of Q is 1·33 ms^{-1}. [3]

The motion is now altered so that Q hangs at rest below O and P moves in a horizontal circle on the table with speed 0·84 ms^{-1}, at a constant distance r from O but tending to slip away from O.

(iii) Find the value of r. [5]

7. A bullet is fired out of a window at a height of 5·2 m above horizontal ground. The initial velocity of the bullet is 392 ms^{-1} at an angle α above the vertical, where $\sin\alpha = \frac{1}{20}$, as shown.



Find

(i) the range of times after firing during which the bullet is 15 m or more above ground level, [5]

(ii) the greatest height above the ground reached by the bullet, [3]

(iii) the horizontal distance travelled by the bullet before it reaches its highest point. [2]

Certain modelling assumptions have been made about the bullet.

(iv) State these assumptions and suggest a way in which the model could be refined. [2]

(v) State, with a reason, whether you think this refinement would make a significant difference to the answers. [2]

MECHANICS 2 (C) TEST PAPER 10 : ANSWERS AND MARK SCHEME

1.	$2m(-1, 5) + 3m(0, 6) + 5m(3, -2) = 10m(\bar{x}, \bar{y})$	$(1\cdot3, 1\cdot8)$	M1 M1 A1 A1	4
2.	$P = (2200 + 5000 \times 0.8)v = 6200 \times 10 = 62$ kW		M1 A1 M1 A1	4
3.	(i) Momentum : $0\cdot05u = M(0\cdot008u)$	$m = 6\cdot25$ kg	M1 A1	
	(ii) K.E. given to bullet $= \frac{1}{2}(0\cdot05)u^2 = \frac{1}{40}u^2$ J		B1	
	K.E. given to gun $= \frac{1}{2}(6\cdot25)(0\cdot008u)^2 = \frac{1}{5000}u^2$ J		M1 A1	
	(iii) $u^2(\frac{1}{40} + \frac{1}{5000}) = 5103$	$u = 450$	M1 A1	7
4.	(i) Resolve vert : $3g = R + \frac{1}{2}S$	Resolve horiz : $\frac{1}{3}R = \frac{\sqrt{3}}{2}S$	M1 A1 A1	
	Hence $3g = \frac{1}{2}(3\sqrt{3} + 1)S$	$S = 6g/(3\sqrt{3} + 1) = 9\cdot49$ N	A1 M1 A1	
	(ii) M(B) : $3g/2 = Sd$ $d = 1\cdot55$ m	$AP = 0\cdot45$ m	M1 A1 A1	9
5.	(i) Momentum : $1\cdot2 - 1\cdot6 = 0\cdot4v_A + 0\cdot8v_B$	$v_A + 2v_B = -1$	M1 A1	
	Elasticity : $(v_B - v_A)/(-2 - 3) = -0\cdot8$	$v_A - v_B = -4$	M1 A1	
	Solve : $v_A = -3,\ v_B = 1$		M1 A1 A1	
	A has speed 3 ms^{-1}, B has speed 1 ms^{-1}, both directions reversed		A1	
	(ii) K.E. lost $= 0\cdot2(9) + 0\cdot4(4) - 0\cdot2(9) - 0\cdot4(1) = 1\cdot2$ J		M1 A1	10
6.	(i) $T = F = \mu R$, so $T = \frac{1}{4}g$	$T\cos\theta = 0\cdot2g$	B1 M1 A1	
	$\cos\theta = 0\cdot8$ $\theta = 36\cdot9°$		A1	
	(ii) $T\sin\theta = 0\cdot2v^2/(0\cdot4\sin\theta)$	$v^2 = 0\cdot5g\sin^2\theta = 1\cdot764$	M1 A1	
	$v = \sqrt{1\cdot764} = 1\cdot33$ ms^{-1}		A1	
	(iii) Now $T = 0\cdot2g$	$0\cdot2g + 0\cdot25g = 0\cdot5v^2/r$	B1 M1 A1	
	$0\cdot45g = 0\cdot5(0\cdot84^2)/r$	$r = 0\cdot08$	M1 A1	12
7.	(i) $y = (392\sin\alpha)t - 4\cdot9t^2 = 19\cdot6t - 4\cdot9t^2$		B1	
	15 m above ground, $y = 9\cdot8$	$19\cdot6t - 4\cdot9t^2 = 9\cdot8$	M1	
	$t^2 - 4t + 2 = 0$ $(t-2)^2 - 2 = 0$	$t = 2 \pm \sqrt{2}$	M1 A1	
	Times are from $0\cdot586$ s to $3\cdot41$ s		A1	
	(ii) y is maximum when $19\cdot6 - 9\cdot8t = 0$	$t = 2$ $y = 19\cdot6$	M1 A1	
	Height above ground $= 24\cdot8$ m		A1	
	(iii) $x = (392\cos\alpha)t = 391\cdot5 \times 2 = 783$ m		M1 A1	
	(iv) Bullet = particle; assumed no air resistance	Include this	B1 B1	
	(v) As bullet is small and moving fast, probably little difference		B1 B1	14